

Comments on matter collineations of plane symmetric, cylindrically symmetric and spherically symmetric spacetimes

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ABSTRACT: Comments are made on some recently published papers on matter collineations of plane symmetric, cylindrically symmetric and spherically symmetric spacetimes.

Recently matter collineations (MCs) of plane symmetric static [1] and cylindrically symmetric static spacetimes [2] have been presented. Earlier, the same author also classified spherically symmetric static spacetimes according to their MCs [3]. For an energy-momentum tensor, \mathbf{T} , we call ξ an MC if

$$\mathcal{L}_\xi \mathbf{T} = 0 . \quad (1)$$

In component form, Eq. (1) becomes the MC equation

$$T_{ab,c}\xi^c + T_{ac}\xi_{,b}^c + T_{bc}\xi_{,a}^c = 0 .$$

In these equations, if \mathbf{T} is replaced by the Ricci tensor, \mathbf{R} , then the vector ξ is called a Ricci collineation (RC). Noting the apparently similar form of the equations and the role of the matter and Ricci tensors in the Einstein field equations (EFEs)

$$R_{ab} - \frac{1}{2}Rg_{ab} = \kappa T_{ab}, \quad (2)$$

the author merely replaced the Ricci tensor by the matter tensor in Refs. [4]-[6] for the classification according to RCs of plane symmetric, cylindrically symmetric and spherically symmetric static spacetimes, respectively. All that remains to be done is to check that the RCs satisfy the MC equations. Even if he has done so, errors persist in his papers. For example, because of the error in calculations the Lie algebra for a case, as given in Eq. (B47) of Ref. [4], does not close. This error has been carried over in Eqs. (43) of his paper [1]. (A typographical error there is carried over as well.) Now, as plane symmetry can locally be considered as a special case of cylindrical symmetry, this particular case appears in Ref. [5] also, where it has been corrected. This correction has been carried over into Eqs. (27) of his paper [2] as well, but the author has not cited these papers!

It is worth mentioning a serious misconception, in the three subject papers [1, 2, 3], that was not imported from the Ricci collineation papers. The author says that there are “three, four, five, six, seven or ten MCs out of which three are isometries and the rest are proper” [1, 2]. He has assumed that the isometry group is minimal. This is simply incorrect as there are numerous cases of non-minimal isometry groups. It is possible that all the four, five, six, seven or ten MCs may be isometries and there may be no proper MCs. A similar problem arises for the spherically symmetric case [3].

We also mention here that Ref. [7] on the same subject as Ref. [1] does not only classify the plane symmetric spacetimes (with the correct Lie algebras provided) but

it discusses the issue of the relationship between the RCs and MCs and provides a number of explicit examples for that purpose also. We will not go into further detail on this because this is a subject of a separate full-length study [8] in itself.

References

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